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# Enhanced mid-infrared transmission through a metallic diffraction grating

**Prabath Hewageegana and Vadym Apalkov**

Department of Physics and Astronomy, Georgia State University,
Atlanta, GA 30303, USA

E-mail: vapalkov@gsu.edu

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**Abstract**
We study theoretically an enhancement of the intensity of mid-infrared light transmitted through a metallic diffraction grating. We show that for s-polarized light the enhancement of the transmitted light is much stronger than for p-polarized light. By tuning the parameters of the diffraction grating, the enhancement of the transmitted light can be increased by a few orders of magnitude. The spatial distribution of the transmitted light is highly nonuniform with very sharp peaks, which have spatial widths of about 10 nm.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

It is well known now that due to the generation of surface plasmons [1, 2] at the boundary between metal and dielectric media the local electromagnetic field can be strongly enhanced [3–11]. The enhancement is observed only near a metal–dielectric interface, away from the interface the effect of the surface plasmons is exponentially suppressed. Due to momentum conservation, the generation of surface plasmons by incident light is prohibited at a translationally invariant metal–dielectric interface. This is because of the mismatch between the momentum of the incident light and that of the surface plasmons. Only at an inhomogeneous metallic surface can the interaction between the surface plasmons and the incident light be observed and the surface plasmons can be generated by the incident light. The inhomogeneity can be introduced, for example, by making the surface spherical, by incorporating subwavelength defects (holes), or by making periodic corrugations, i.e. a grating, on a flat metallic surface.

There are many different applications, not only in physics but also in chemistry and biology, of the surface plasmon enhancement of electromagnetic waves. One of them is a strong signal enhancement in the surface-enhanced Raman spectroscopy experiments [12, 13]. Here a molecule is placed near the metallic nanostructure, and due to interaction of the incident light with localized surface plasmons the intensity of the electromagnetic field near the molecule becomes strongly enhanced. This enhancement produces a strong interaction of incident light with the molecule. In these experiments the coupling of the incident light to the surface plasmons is achieved due to the nanoscale size of the metallic system.

Another method, which is widely used to generate the surface plasmons at a metal–dielectric interface, is based on a diffraction grating placed on top of a plasmon waveguide [2, 14, 15]. In this case, a grating vector provides the momentum required by the momentum conservation. The diffraction grating has been widely used to generate not only the surface plasmons, but also the plasmon excitations in low dimensional semiconductor systems [16–18].

A strong plasmonic effect and the corresponding strong plasmonic enhancement is observed for near-infrared light only. A corresponding metallic structure and a spatial region of the enhancement of electromagnetic field has a nanoscale size. At the same time there are many systems that are sensitive to mid-infrared light. Examples of such systems are the intersubband quantum well and quantum dot photodetectors [19, 20] or an isolated quantum dot or molecular clusters, where optical transitions correspond to an energy of 200–400 meV. In this relation the question can be asked: can a metallic structure introduce the local enhancement of an electromagnetic field in the mid-infrared frequency region? To study this problem we consider a metallic grating on a metal–dielectric interface. The incident light can be p- or s-polarized. For p-polarized light, where magnetic field is parallel to the metal–dielectric interface, the enhancement of the light is due to the generation of surface plasmons at the metal–dielectric interface. Similarly to other applications of surface plasmons, we should expect a local enhancement of

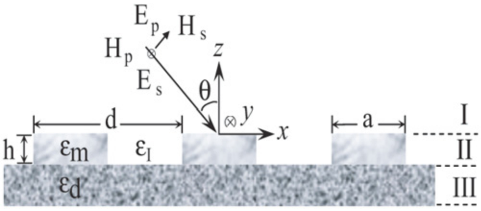

**Figure 1.** Schematic illustration of the metal grating on the surface of dielectric media. Here, the grating period is $d$ and the grating height is $h$. The grating consists of periodic strips of metal with dielectric constant $\varepsilon_{\mathrm{m}}(\omega)$ and air with dielectric constant $\varepsilon_{\mathrm{I}} = 1$. The width of metallic strips is $a$. The region III is filled by a material with dielectric constant $\varepsilon_{\mathrm{d}}$. Here **E** is the electric field vector and **H** is the magnetic field vector. The angle $\theta$ is the incident angle.

the electromagnetic field near the metal surface. The metallic diffraction grating also opens the possibility of enhancing the intensity of the s-polarized incident light (electric field is parallel to the metal–dielectric interface). Since the surface plasmons are sensitive to the normal component of the electric field only, which is zero in the case of the s-polarized light, then the modes of the diffraction grating for the s-polarized light are not the surface plasmons. In this case the possible enhancement of the s-polarized light is due to generation of the local modes of the diffraction grating, which are not related to the surface plasmons. Below we consider both s- and p-polarizations and study the enhancement of the electromagnetic field near the metal–dielectric interface.

The effect of the metal grid gate on the absorption efficiency of the terahertz electromagnetic radiation has been studied recently both theoretically and experimentally [21–25]. The metal grating in [21, 22] was responsible for the coupling of the incident light to the plasmon excitations within a double quantum well system of a field-effect transistor. In our case the incident light is coupled to the local modes of the diffraction grating. This coupling results in redistribution of the electromagnetic field intensity, which produces the local enhancement of the incident light.

## 2. Main system of equations

The system under consideration consists of three regions: region I (air) with dielectric constant $\varepsilon_{\mathrm{I}} = 1$, region II (metallic diffraction grating with period $d$ and height $h$), and region III (dielectric) with dielectric constant $\varepsilon_{\mathrm{d}}$. The system is shown schematically in figure 1. We assume that the dielectric constant of metal, $\varepsilon_{\mathrm{m}}(\omega)$, has a Drude type dependence on the frequency

$$\varepsilon_{\mathrm{m}}(\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega(\omega + i/\tau)}, \tag{1}$$

where $\omega_{\mathrm{p}}$ is the plasma frequency and $\tau$ is the phenomenological relaxation time. Below we assume that the metal is gold and $\omega_{\mathrm{p}} = 3.39 \times 10^{15}\ \mathrm{s}^{-1}$ and $\tau = 1.075 \times 10^{-14}$ s [26].

To describe the distribution of the electromagnetic field we introduce the coordinate system with axis $z$ orthogonal to the metal–dielectric interface, and axis $x$ in the plane of the interface, see figure 1. The direction of the incident light is characterized by an incident angle $\theta$. The incident light can have two polarizations: s- and p-polarizations. For the p-polarized light, the magnetic field of the electromagnetic wave is in the plane of the interface, while for the s-polarized light the electric field of the wave is in the interface plane.

To find the distribution of electromagnetic field we need to solve the system of Maxwell's equations with the corresponding boundary conditions. Namely, the amplitude of the incident light (at $z > 0$) is given and there are no waves incident on the diffraction grating outside of the grating region at $z < -h$, i.e. in this region there are only outgoing waves. The Maxwell equations for s- and p-polarized lights become decoupled and electric and magnetic fields can be described in terms of a single function: $\psi = E_y$ in the case of the s-polarized light and $\psi = \sqrt{\mu_0/\epsilon_0}H_y$ in the case of the p-polarized light [27].

To solve the corresponding Maxwell's equations we used the well known *modal expansion method* [28]. In this method the solutions of the Maxwell's equations are expressed in terms of the eigenmodes of electromagnetic field in all three regions. In regions I and III these eigenmodes are simple plane waves [27]. The incident light has an $x$ component of the wavevector equal to

$$k_x = k \sin\theta, \tag{2}$$

where $k = \omega/c$ is the wavevector of the incident light. The diffraction grating with a period $d$ introduces a coupling of the incident light with the plane waves, the $x$ components of the wavevectors, $k_{xn}$, which are given by the expression

$$k_{xn} = k\left[\sin\theta + n\lambda/d\right], \tag{3}$$

where $n = 0, \pm1, \pm2, \ldots$ is a diffraction order, and $\lambda$ is the wavelength of the incident light.

The general solution of Maxwell's equations in region II can be expressed in terms of the eigenmodes of the diffraction grating. The eigenmodes of the wave equation in the grating region are characterized by parameter $\kappa_z$, which satisfies the following nonlinear equation [27].

$$\frac{\xi_j^2 + 1}{2\xi_j} \sin[\beta_1 a] \sin[\beta_2 a] - \cos[\beta_1 a] \cos[\beta_2 a] = -\cos(k_x d). \tag{4}$$

Here $j = \mathrm{p}, \mathrm{s}$, where p stands for the p-polarization and s stands for the s-polarization. We also introduce the following notations

$$\begin{aligned} \xi_{\mathrm{p}} &= [\varepsilon_{\mathrm{m}}(\omega)\beta_2/\varepsilon_{\mathrm{I}}\beta_1], & \xi_{\mathrm{s}} &= [\beta_2/\beta_1], \\ \beta_1 &= \sqrt{\varepsilon_{\mathrm{m}}k^2 - \kappa_z^2}, & \beta_2 &= \sqrt{\varepsilon_{\mathrm{I}}k^2 - \kappa_z^2}. \end{aligned} \tag{5}$$

The solution of equation (4) determines an infinite number of eigenmodes. We can enumerate them by index $\ell$, i.e. $\kappa_{z\ell}$ is the $\ell$th solution of equation (4).

Since the frequency range considered in the present paper corresponds to a very large magnitude of the metal dielectric constant ($\varepsilon_{\mathrm{m}} \approx -847 + 1127i$), the numerical procedure of finding the eigenmodes of the diffraction grating becomes highly unstable. Usually the direct numerical solution of nonlinear eigenequation (4) does not provide all the eigenmodes of the diffraction grating, i.e. some of the eigenmodes can be overlooked. To resolve this problem we applied the method of tracing the root trajectory [29], i.e. we found the roots of eigenequation (4) by tracing the root trajectory in the complex plane of dielectric constant [29]. Namely, first we calculated all necessary roots of equation (4) at small value of metal dielectric constant. Then we introduced a straight line in the complex plane of the dielectric constant. This line connects the initial small dielectric constant and the final large dielectric constant. Finally, we find the root trajectory along the straight line in the complex dielectric plane, i.e. we trace the values of the roots when the dielectric constant is changed along the line [29]. With this method we can find all necessary eigenmodes of the diffraction grating including the hidden modes for the p-polarized light [30].

Finally, imposing the continuous boundary conditions between different regions, we obtain the system of linear equations from which we can find the amplitudes of eigenmodes in regions I, II, and III. To make the system finite we introduce the maximum value of the diffraction order, $n_{\max}$, so that $|n| \leqslant n_{\max}$. The value of $n_{\max}$ determines also the maximum number of eigenmodes of the diffraction grating. Namely, $\ell \leqslant (2n_{\max}+1)$. Therefore the final size of the system is $2(2n_{\max}+1)$. The values of $n_{\max}$ are found from the condition of convergence of reflection, transmission, and absorption coefficients. Within the range of parameters considered in the present paper, we have found that the method converges at $n_{\max} \approx 15$ for the p-polarized light and at $n_{\max} \approx 30$ for the s-polarized light. Therefore, throughout the paper we use $n_{\max} = 25$ and $n_{\max} = 50$ for p-polarization and s-polarization, respectively. With these values of $n_{\max}$ we solved the corresponding system of linear equations numerically.

The intensity of the electromagnetic waves in region III is described by the following equation

$$I(x,z) = \left|\psi^{(\mathrm{III})}(z,x)\right|^2 \tag{6}$$

for the s-polarized light, which has only an in-plane ($y$) component of electric field, and

$$I(x,z) = \left|\partial_x \psi^{(\mathrm{III})}(z,x)\right|^2 \tag{7}$$

for the p-polarized light. Although for the p-polarized light there are both a $z$ component and an in-plane ($x$) component of electric field, in equation (7) we take into account only the $z$ component of electric field. In terms of the possible applications, it means that for p-polarized light we study only optical transitions due to the perpendicular component of electric field. Below we also consider the average intensity of the electromagnetic waves, which is given by the expression

$$I_{\mathrm{av}}(z) = \frac{1}{d}\int_0^d I(x,z)\,\mathrm{d}x. \tag{8}$$

To characterize the enhancement of the transmitted light due to the presence of the diffraction grating, we measure the intensity $I(x,z)$ in units of an intensity, $I_0$, of an electromagnetic wave in the region III without a diffraction grating. The intensity $I_0$ is given by the standard Fresnel's equations. Namely,

$$I_0 = \frac{4k_z^2}{[k_z + k_z^{(\mathrm{III})}]^2} \tag{9}$$

for the s-polarization and

$$I_0 = \frac{4k_z^2 \sin^2\theta}{\varepsilon_{\mathrm{d}}[\varepsilon_{\mathrm{d}} k_z + k_z^{(\mathrm{III})}]^2} \tag{10}$$

for the p-polarization [31]. Here $k_z = k\cos\theta$. Therefore, if $I(x,z)$ is greater than 1 (in units of $I_0$) then there is enhancement of the transmitted light due to the presence of the diffraction grating.

## 3. Results and discussion

### 3.1. Average intensity

As we have mentioned in section 1 we expect an enhancement of the intensity of the light transmitted through the diffraction grating. This enhancement is due to generation of the local modes of the diffraction grating. Usually, when such a type of enhancement is discussed, it is due to generation of the surface plasmons at the metal–dielectric interface. In the present problem the surface plasmons are coupled only to the p-polarized incident light, while for the s-polarized light the incident light is coupled to the general modes of the diffraction grating. Therefore in general we should expect much stronger enhancement for the p-polarized light. It happens that in the present problem the enhancement of the incident light is much stronger for the s-polarized light. This is mainly because we consider a low-frequency range for the incident light, within which the dielectric constant of the metal is relatively large. For the parameters of the system considered in the present paper the dielectric constant of the metal is $\approx -847 + 1127\mathrm{i}$.

At first we analyze the average intensity (see equation (8)) of the transmitted light. There are a few parameters of the diffraction grating that determine the properties of the average intensity. These parameters are the period of the diffraction grating, $d$, the metallic coverage of the dielectric media, $a/d$, the incident angle, $\theta$, and the height, $h$, of the diffraction grating.

As a function of frequency of the incident light the average intensity has a maximum. This maximum corresponds to the resonant condition, when the frequency of the incident light is in resonance with the frequency of the diffraction grating modes. In figure 2 the results of calculations are shown for s- and p-polarized light and for different metallic coverage, $a/d$, of the dielectric media. We can clearly see the difference between the s- and p-polarized light. For the p-polarization we have a very sharp maximum (panels (b) and (c)). The position of the maximum has weak dependence on the metallic coverage, $a/d$. This is an indication that the incident light is coupled to the surface plasmons, since for the surface plasmons the energy of the generated plasmon depends on its wavevector, which is determined by $\pi/d$ only. Therefore, there is no dependence of the wavevector of the plasmon excitation within

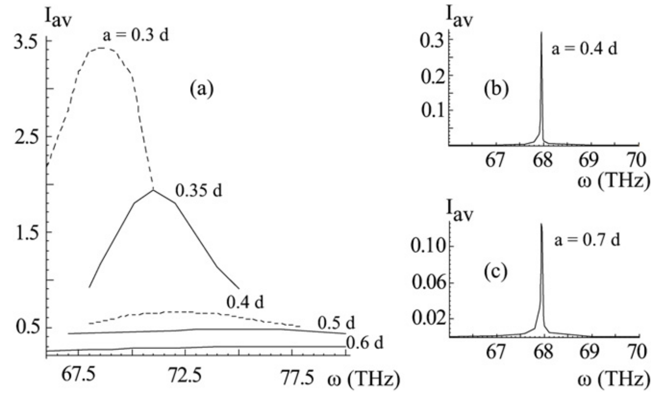

**Figure 2.** (a) Calculated average intensity, $I_{\mathrm{av}}$, for s-polarized light at a distance 10 nm below the grating (i.e. in region III) in units of $I_0$ for different $a/d$ values, as indicated in the panel. (b) The same plot for p-polarized light for $a = 0.4d$ and (c) for $a = 0.7d$. For all panels the grating period $d = 2\ \mu$m, the incident angle $\theta = 45°$, and the height of the diffraction grating $h = 50$ nm.

the diffraction grating on the parameter $a/d$. At the same time the average intensity, even at the maximum, is less than 1 (in units of $I_0$). This means that, on average, there is no enhancement of the intensity of the transmitted light due to the diffraction grating. The results in figure 2 are shown for the incident angle $\theta = 45°$. Below we show that at small incident angles there is some enhancement of the transmitted light.

A completely different behavior is observed for the s-polarized light, see figure 2(a). The maximum now is broad and there is a strong dependence of the position of the maximum, $\omega_{\max}$, and the maximum intensity on the value of $a/d$. With decreasing metallic coverage, i.e. $a/d$, the line becomes red shifted, and the maximum intensity increases. For example, if $a/d$ decreases from 0.5 to 0.3 then the maximum intensity increases six times and becomes 3.5. Since at $a = 0$ the average intensity should be 1 (in units of $I_0$), then at small values of $a$ we should expect a decrease of the maximum intensity with decreasing $a/d$. This means that there is an optimal value of $a/d$ for which the maximum intensity is the largest. Due to numerical instability of the problem we cannot go below $a/d = 0.3$ and find the optimal value of $a/d$.

The reason why the maxima in figure 2(a) is broad and we have a strong dependence on $a/d$ is that in the case of s-polarization the incident light is coupled not to the surface plasmons but to the modes of the diffraction grating. Such modes depend on the actual structure of the diffraction grating, i.e. on the value of $a/d$.

The dependence of the position of the peak, $\omega_{\mathrm{peak}}$, of the average intensity on the metallic coverage, $a/d$, is shown in figure 3. We can see that the frequency $\omega_{\mathrm{peak}}$ does not depend on $a/d$ for the p-polarized light, while there is a strong dependence of $\omega_{\mathrm{peak}}$ on $a/d$ for the s-polarization. This behavior supports the structure of the modes of the diffraction grating for the different polarizations of the light we discussed above. From figure 3 we can conclude that only for s-polarization can we use $a/d$ as a tuning parameter, i.e. by changing $a/d$ we can change the position of the maximum of $I_{\mathrm{av}}$.

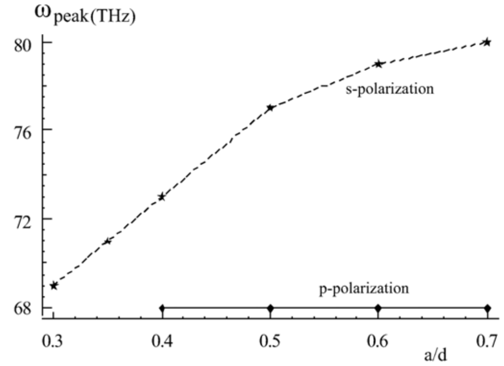

**Figure 3.** The peak frequency, $\omega_{\mathrm{peak}}$, at the maximum value of average intensity is shown as a function of $a/d$ for both s- and p-polarization. The grating period $d = 2\ \mu$m, the incident angle $\theta = 45°$, and the height of the diffraction grating $h = 50$ nm.

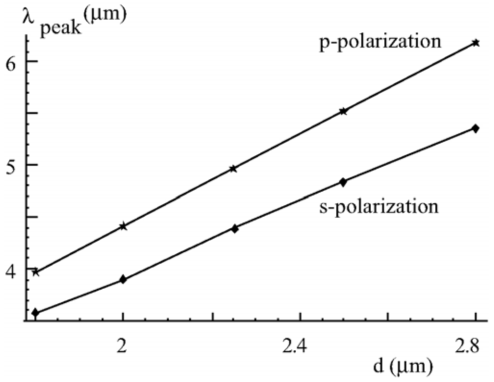

**Figure 4.** The peak wavelength, $\lambda_{\mathrm{peak}}$, is shown as a function of grating period $d$, for both p- and s-polarizations. The ratio $a/d = 0.5$ and the incident angle $\theta = 45°$.

The position of the maxima of the average intensity depends on the period, $d$, of the diffraction grating for both p- and s-polarized light. These dependencies are shown in figure 4 and in terms of the wavelength of incident light can be approximately described by the linear functions. Namely,

$$\lambda_{\mathrm{peak}} = 1.8d + 0.31 \tag{11}$$

for s-polarization and

$$\lambda_{\mathrm{peak}} = 2.2d - 0.0047 \tag{12}$$

for p-polarization. For p-polarization this linear dependence is universal, i.e. it does not depend on the parameter $a/d$ and on the incident angle (as we will see below). For s-polarization the coefficients in equation (11) depend on the value of $a/d$. Another difference between s- and p-polarizations is that the peak wavelength of the s-polarized light is less than the peak wavelength of the p-polarized light. This is due to a different nature of the eigenmodes of the diffraction grating for the different polarizations of the light.

The slope of the $\lambda_{\mathrm{peak}}(d)$ dependence for the p-polarized light is 2.2 (see equation (11)). This value is very close to the value obtained experimentally in the terahertz frequency

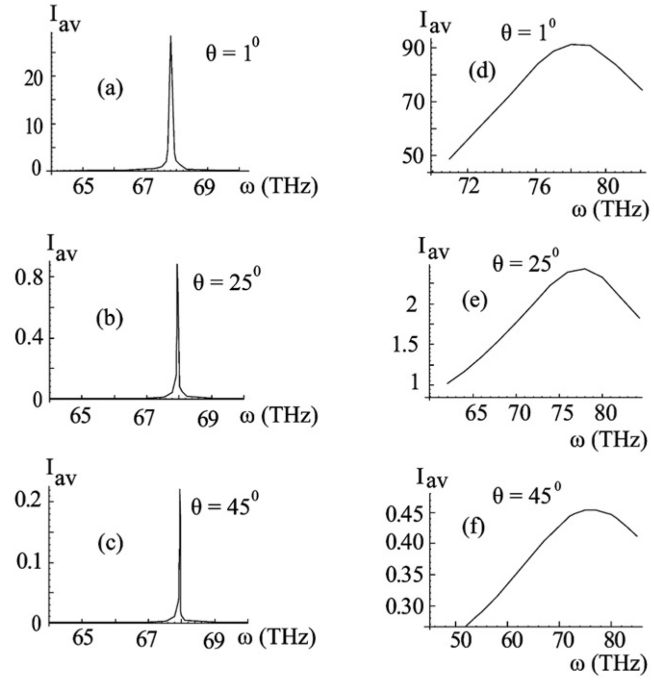

**Figure 5.** The average intensity, $I_{\mathrm{av}}$, at distance $z = 50$ nm below the grating (in region III) in units of $I_0$ for different incident angle $\theta$ as indicated in the panels. The panels (a)–(c) correspond to p-polarization and (d)–(f) correspond to s-polarization. For all the panels the grating period $d = 2$ $\mu$m and the ratio $a/d$ is 0.5.

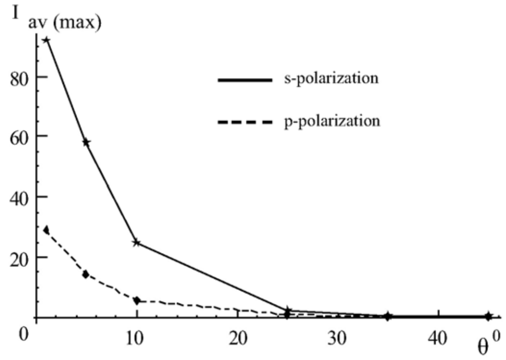

**Figure 6.** The maximum average intensity (peak value), $I_{\mathrm{av(max)}}$, for the p- and s-polarized light at distance $z = 10$ nm below the grating (i.e. in region III) is shown as a function of incident angle, $\theta$. The intensity is in units of $I_0$. The grating period $d = 2$ $\mu$m, the ratio $a/d = 0.5$, and the height of the diffraction grating $h = 50$ nm.

range [32]. The experimental result for the slope is 2.19. Although the experimental data [32] have been obtained at terahertz frequency, i.e. ≈30 THz, and our results are for ≈70 THz, the fact that the slope is almost the same indicates that at these frequencies the position of the maxima of the average intensity does not depend on the parameters of the metal, i.e. on the dielectric constant of the metal. This fact also indicates that the nature of the diffraction grating modes, which are responsible for the intensity maxima of the p-polarized light, is the same at all terahertz frequency ranges. These modes are the surface plasmons.

In figures 5 and 6 we show the dependence of the average intensity on the incident angle, $\theta$. We can see that for both p- and s-polarizations the position of the maximum has a very weak dependence on the incident angle. This means that the eigenfrequencies of the modes of the diffraction grating have weak dependence on $\theta$. At the same time the maximum intensity strongly depends on the angle, $\theta$. We can see that with decreasing the incident angle the maximum intensity increases and at small angles it can be much large than 1. This means that at small incident angle the diffraction grating provides strong enhancement of the intensity of the transmitted light. For the s-polarization the diffraction grating can enhance the intensity of the transmitted light by almost two orders of magnitude. At all incident angles the enhancement of the electromagnetic wave is much stronger for s-polarization than for p-polarization.

Therefore we expect a strong enhancement of intensity of transmitted light at small incident angle. The enhancement is the strongest for the s-polarized light. The behavior of the enhancement as a function of the incident angle for the p-polarized light is unexpected. The modes, which are responsible for the enhancements of the p-polarized light are the surface plasmons. At the same time we know that the coupling of the light to the surface plasmons is determined by the component of the electric field, which is orthogonal to the metal–dielectric interface. Therefore we should expect that if we increase the normal component of electric field then the generation of the surface plasmons and the corresponding enhancement of the transmitted light is increased. But for the p-polarized light we see in figure 6 the opposite tendency is observed: with decreasing the angle, i.e. decreasing the $z$ component of electric field, the enhancement is increased. This fact shows that for the p-polarized light the modes of the diffraction grating are not purely surface plasmons. These modes contain both the normal and in-plane components of the electric field. We still identify these modes as the surface plasmon modes since, as we will see below, the light in these modes is mainly localized near the metal–dielectric interface.

As we can see from figure 6 the enhancement, i.e. the average intensity of the transmitted light in units of $I_0$, increases with decreasing the incident angle. At the same time, for the p-polarized light the intensity $I_0$ has a strong dependence on the incident angle, $\theta$, itself. Namely, the intensity $I_0$ decreases with decreasing $\theta$, see equation (10). Therefore there is an optimal value of the incident angle, $\theta$, which is determined by the condition that the intensity (not the enhancement) of the transmitted light is maximum. To find the actual intensity of the transmitted light we just need to multiply the data shown in figure 6 by $I_0(\theta)$. In figure 7 we show the dependence of the intensity of transmitted light on the incident angle for p-and s-polarized light. For s-polarized light we have a monotonic dependence on the incident angle and the maximum intensity of the transmitted light is achieved for the normal incident light. For the p-polarized light we can clearly see that there is an optimal value of the incident angle. This angle is around $\theta \approx 5°$. We can also see a general tendency that the intensity of the s-polarized transmitted light is much larger than the intensity of the p-polarized transmitted light.

Another parameter of the diffraction grating, which can be varied to optimize an enhancement of the transmitted light,

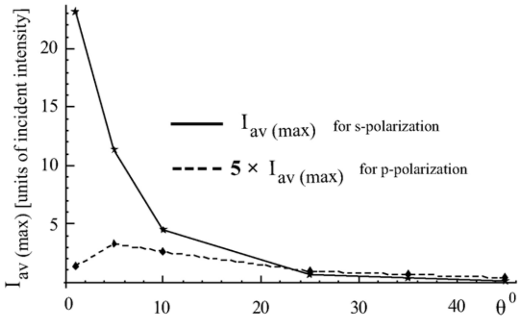

**Figure 7.** The maximum average intensity (peak value), $I_{\mathrm{av(max)}}$, for the p-polarized light (scaled by a factor of 5) and s-polarized light at distance $z = 10$ nm below the grating (in the region III) is shown as a function of incident angle $\theta$. The intensity is in the units of the intensity of the incident light. The grating period $d = 2\ \mu$m, the ratio $a/d = 0.5$, and the height of the diffraction grating $h = 50$ nm.

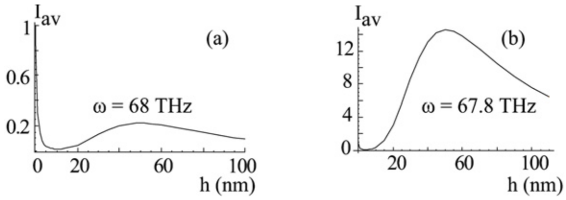

**Figure 8.** (a) The average intensity, $I_{\mathrm{av}}$, for p-polarized light at distance $z = 50$ nm below the grating (in the region III) is shown in units of $I_0$ as a function of $h$ for $a/d = 0.5$ (a) at $\omega = 68$ THz for $\theta = 45^\circ$ and (b) at $\omega = 67.8$ THz for $\theta = 5^\circ$. For all the panels the grating period $d = 2\ \mu$m.

is the height of the grating. When the height of the grating is large then the intensity of the transmitted wave should be small and $I_{\mathrm{av}} \approx 0$. At zero height, i.e. without the diffraction grating, the average intensity is 1 (in units of $I_0$). Therefore, if at intermediate values of $h$ there is an enhancement of the intensity of the transmitted light, i.e. $I_{\mathrm{av}} > 1$, then the dependence of $I_{\mathrm{av}}$ on $h$ is nonmonotonic and at some value of $h$ we should see the maximum of the average intensity. This is the optimal value of the height of the diffraction grating.

In figure 8 we show the dependence of the average intensity on the height, $h$, of the diffraction grating for the p-polarized light. The specific feature of this dependence is that it is nonmonotonic even for the parameters of the system, for which the average intensity is less than 1 at all values of $h$. For example (see figure 8(a)), at the incident angle $\theta = 45^\circ$ the average intensity is less than 1 at all $h$, but there is a local maximum at $h \approx 50$ nm. At smaller incident angle, e.g. $\theta = 5^\circ$ (see figure 8(b)), there is an enhancement of the transmitted light and we have an absolute maximum at $h \approx 50$ nm. The position of the absolute maximum determines the optimal value of the height of the diffraction grating. We can also notice the local minimum at small $h$, $h \approx 8$ nm.

A different behavior is observed for the s-polarized light, see figure 9. In this case if the average intensity is less than 1 then the dependence of $I_{\mathrm{av}}$ on $h$ is monotonic, i.e. there is no local maximum, which is different from p-polarization (see figure 8). The typical dependence is shown in figure 9(a), where for $a/d = 0.5$ the average intensity is less than 1. If the average intensity is greater than 1, then there is an absolute maximum at some finite value of $h$. For example, at $a/d = 0.3$ the maximum of the average intensity is achieved at $h \approx 55$ nm.

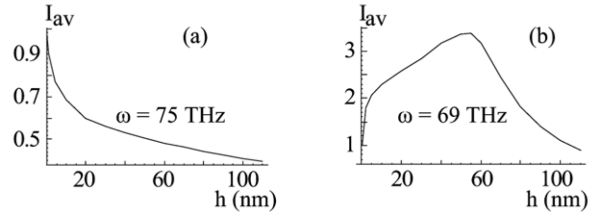

**Figure 9.** The average intensity, $I_{\mathrm{av}}$, for s-polarized light at distance $z = 50$ nm below the grating (i.e. in the region III) is shown in units of $I_0$ as a function of $h$ (a) at $\omega = 75$ THz and $a/d = 0.5$ and (b) at $\omega = 69$ THz and for $a/d = 0.3$. For all the panels the grating period $d = 2\ \mu$m and the incident angle $\theta = 45^\circ$.

Finally, we can summarize the properties of the average intensity of the transmitted light for s- and p-polarizations. From the data shown above we can conclude that the enhancement of the transmitted light is the largest for the s-polarized light. For s-polarization, the frequency position of the intensity maxima can be tuned by varying the period of the diffraction grating, $d$, and the metallic coverage of the dielectric media, $a/d$. For the p-polarized light only the period of the diffraction grating affects the frequency position of the maximum. For both p- and s-polarized light there is no dependence of the position of the intensity maxima on the incident angle.

It is possible to find the optimal parameters of the diffraction grating so that the enhancement of the transmitted intensity is the largest. For the s-polarized light the decrease of the metallic coverage, $a/d$, and the incident angle, $\theta$, results in an increase of the average intensity, while for the p-polarized light only the incident angle affects the intensity of transmitted light. As a function of the height of the diffraction grating, the average intensity has a maximum for both p- and s-polarizations. For the parameters of the system considered in the present paper the maximum is achieved at $h \approx 50$ nm.

### *3.2. Spatial intensity distribution and the modes of the diffraction grating*

To better understand the structure of the modes of the diffraction grating, which are responsible for the enhancement of the transmitted light, we analyze a spatial distribution of an intensity of the transmitted light in region III (see figure 1). We expect to see bright spots, i.e. small regions with high intensity, in this distribution. The spatial distribution of the transmitted electromagnetic field in region III is shown in figure 10 for the p-polarized light. The main tendency, which we observe in this figure, is that the intensity of the transmitted wave is nonuniform with the maxima at points $x = nd$ and $a + nd$, i.e. at the points corresponding to the boundaries between metallic strips and air. With increasing enhancement of the transmitted light, i.e. with increasing average intensity, the peaks become sharper and more localized. The spatial intensity

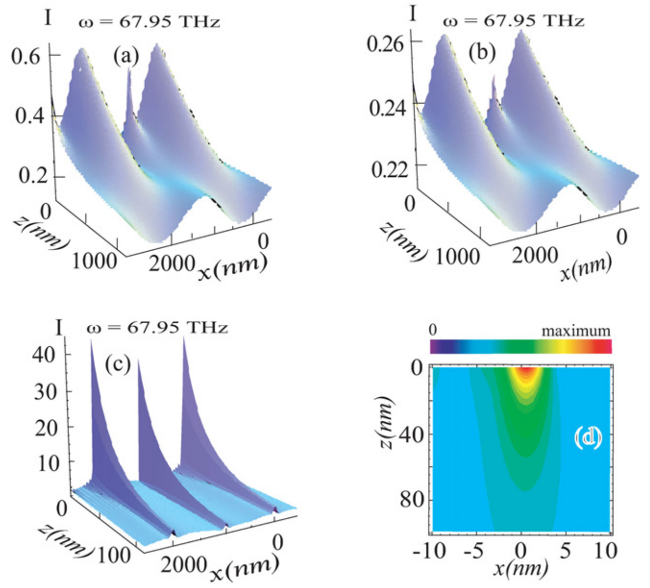

**Figure 10.** The distribution of electric field intensity for p-polarized light is shown in region III. (a) Intensity $I(x, z)$ at peak frequency, $\omega = 67.95$ THz, for $\theta = 45°$ and $a = 0.3d$. (b) The same plot for $a = 0.5d$. (c) Similar plot for $\theta = 5°$ and $a = 0.5d$. (d) Electric field intensity in the vicinity of the first peak in (c) is shown in the $x$–$z$ plane. The scale of the intensity is indicated by the color (online only) bar at the top. For all the panels the grating period $d = 2$ $\mu$m and the height of the diffraction grating $h = 50$ nm.

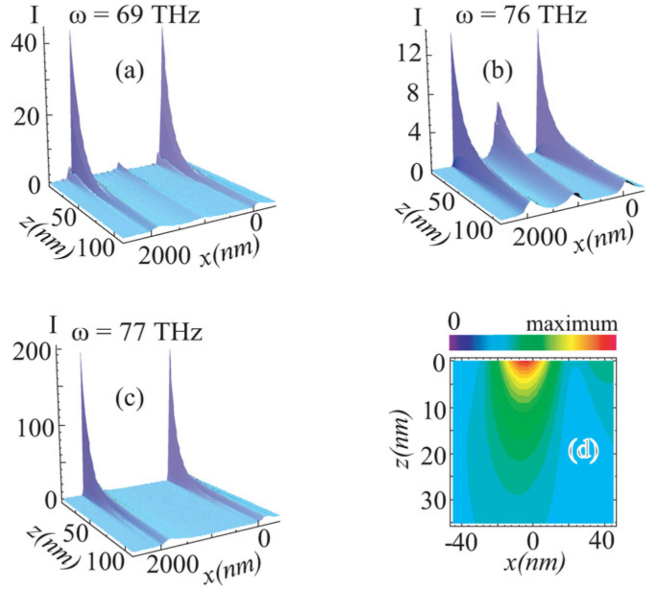

**Figure 11.** The distribution of electric field intensity for s-polarized light is shown in region III. (a) $\theta = 45°$ and $a = 0.4d$; (b) $\theta = 45°$ and $a = 0.5d$; (c) $\theta = 5°$ and $a = 0.5d$. (d) Electric field intensity in the vicinity of the first peak in (c) is shown in the $x$–$z$ plane. The scale of the intensity is indicated by the color (online only) bar at the top. For all the panels the grating period $d = 2$ $\mu$m and the height of the diffraction grating $h = 50$ nm.

distribution has a weak dependence on metallic coverage, $a/d$ (see figures 10(a) and (b)). At large incident angles, i.e. at small values of $I_{\mathrm{av}}$, the peaks become broad, and the contrasts of the peaks become small, i.e. the ratio of the maximum intensity and the minimum intensity is around 2. For the parameters of the diffraction grating, which correspond to the enhancement of the average intensity, the peaks become very sharp (see figure 10(c)). In this case the widths of the peaks are around 10 nm and the contrasts of the peaks are large. For example, in figure 10(c) the ratio of the maximum value of the intensity to the minimum value is around 40.

In figure 11 we show a spatial intensity distribution for the s-polarized light. In contrast to the p-polarized light, we have much stronger local enhancement of the intensity of the transmitted waves. Another difference from the p-polarization is that now the bright spots are located at the points corresponding to only one side of the metal–air interface, i.e. at $x = nd$, while at points $x = a + nd$ the enhancement is much weaker. The contrast of the bright spots for the s-polarization can be up to 200. The intensity at the peak decreases with the distance from the metal–dielectric interface, and the characteristic length is around 20 nm. The widths of the peaks are the same as for the p-polarized light and they are around 10–20 nm. Similar to the p-polarized light, the peaks become more localized when the average intensity is increased. Compared to the average value of the intensity we can say that the intensity at the peak can be 10–100 times larger than the average value.

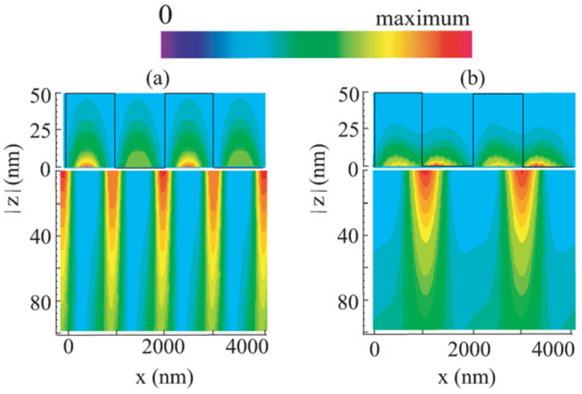

**Figure 12.** Distribution of $I(x, y)$ is shown in the grating region (region II) and below the grating region (region III) for (a) p-polarization and (b) s-polarization. For all the panels: $d = 2$ $\mu$m, $a/d = 0.5$, $\theta = 10°$, and $h = 50$ nm.

To clarify the difference between p- and s-polarized light we show in figure 12 the spatial distribution of the intensity of electric field in the grating and dielectric regions (regions II and III). We can see that the peaks for the p-polarized light are more localized in the $x$-direction than the corresponding peaks for the s-polarized light. At the same time for the s-polarization the peaks are more extended in the $z$-direction. Therefore, if we need to use the advantage of the bright spots in the transmitted waves then, for the s-polarization there are more restrictions on the positions of the sensitive elements (for example, quantum dot) in the $z$-direction. Such elements should be placed at a distance from the metal–dielectric interface of less than 30–40 nm.

Another crucial difference between p- and s-polarized light is that the number of bright spots for the p-polarization is twice as large as the number of bright spots for the s-polarization. This follows from the fact (see figure 12) that the bright spots for the s-polarized light appear only at one side of the metal–air interface. This is due to the finite value of the incident angle, i.e. the angle between the direction of the incident light and the normal to the metal–dielectric interface. However, in the case of the normal incidence ($\theta = 0$) the system becomes symmetric and the bright spots will be on both sides of the metal–dielectric interface.

The behavior of the waves inside the diffraction region is also different for p- and s-polarized light. We can see that the electromagnetic field is mainly localized inside the metal region for the p-polarized light and inside the air region (between metallic strips) for the s-polarized light. In the $z$-direction the electromagnetic field is localized in both cases at the interface between regions II and III. Based on this behavior of the field inside the diffraction grating we can say that the modes, which are responsible for the enhancements of the incident light, are the surface plasmon modes for the p-polarized light and the modes trapped in the air region between two metallic strips for the s-polarized light. For the s-polarization, the dependence of the energy of the trapped modes on the width of the air region, i.e. on the value of $(d - a)$, explains the dependence of the average intensity of the transmitted light on the ratio $a/d$ (see figure 2(a)). We can see from the figure that with decreasing $a/d$ the frequency position of the intensity maximum decreases. This is because with decreasing $a$ the width, $(d - a)$, of the air regions increases. Then the size of the trapped mode also increases and correspondingly the 'energy' of this mode decreases. With increasing size of the trapped mode, the coupling of the incident light with the mode is increased, which results in an increase of the maximum intensity, as shown in figure 2(a).

## 4. Conclusion

We have found in the present work that the enhancement of the electromagnetic field due to the presence of the diffraction grating is much stronger for the s-polarized light than for the p-polarized light. This is opposite to what we should expect if we assume that the enhancement of the light is due to generation of surface plasmons, since the surface plasmons can be generated only by the p-polarized light, but not by the s-polarized light. The manifestation of the surface plasmons is not so strong in our system since we are working in the low-frequency range, where the coupling of the incident light to the surface plasmons is weak.

For the s-polarized light the enhancement of the transmitted light is due to generation of the modes of the electromagnetic field, which are trapped between the metallic strips in the diffraction grating region. For the p-polarized light, the modes responsible for the light enhancement are the surface plasmon modes. As a result, the light in the diffraction grating is localized in the metallic region for the p-polarized light and in the air region for the s-polarized light.

The dependence of the enhancement of the transmitted light on the parameters of the diffraction grating can be summarized as follows.

- Decreasing the metallic coverage, $a/d$, of the dielectric media increases the intensity of the s-polarized light, but does not affect the p-polarized light. For the s-polarized light there is an optimal value of $a/d$ for which the enhancement is maximum.
- Decreasing the incident angle increases the enhancement of the light for both p- and s-polarizations. At the same time for both polarizations the incident angle does not change the frequency positions of the intensity maxima. For the actual value of the intensity of the transmitted light there is an optimal incident angle, $\theta \approx 5°$, for the p-polarized light, at which the intensity of the transmitted light is maximum. For s-polarized light the actual intensity of the transmitted light has a monotonic dependence on the incident angle and the maximum of intensity is realized at zero incident angle.
- The intensity of the transmitted light for both p- and s-polarizations has nonmonotonic dependence on the height, $h$, of the diffraction grating. There is an optimal value of $h$, at which the intensity has a maximum. This value is around 50 nm for the parameters of the system considered in the present paper.
- The intensity of the transmitted wave decreases with the distance from the metal–dielectric interface. The characteristic length is around 50 nm.
- The spatial distribution of the intensity of transmitted light is highly nonuniform with sharp peaks. The spatial size of the peaks is around 10 nm. For the s-polarized light the intensity at the peak is two orders of magnitude larger than the average intensity.

Therefore, to have the largest enhancement of the transmitted light we need to use the s-polarized light. In addition, the metallic coverage, $a/d$, and the incident angle should be small. The height of the diffraction grating should have the optimal value, which is around 50 nm.

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